C. Verification of inequality (28) in Theorem 4.2

For notational simplicity, $\widetilde{\boldsymbol{x}}$ and $\widehat{\boldsymbol{x}}$ denote the optimal allocations with respect to $(\widetilde{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})$ and $(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})$, respectively.

Consider the allocations of all EVs at time t_1 with respect to the bid profile $(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})_{t_1}$. Because $\widehat{d}_{nt_1} < \widehat{d}^*_{nt_1} < d^*_{nt_1} < \widehat{d}^2_{nt_1}$, (18a) gives,

$$\widehat{\beta}_{nt_1} \triangleq \beta_{nt_1}(\widehat{d}_{nt_1}; A) > \beta_{nt_1}(\widehat{d}^2_{nt_1}; A) = \beta^*_{nt_1}.$$

Also, by Lemma 3.2, $\beta^*_{nt} \geq \beta^*_{mt}$ for all $m \in \mathcal{N} \backslash \{n\}$ when $d^*_{nt} > 0$. Therefore, $\widehat{\beta}_{nt_1} > \beta^*_{mt_1}$. Using an argument similar to that following (43), it is straightforward to show,

$$\widehat{x}_{nt_1} = \widehat{d}_{nt_1}, \qquad \widehat{x}_{mt_1} = d^*_{mt_1} \text{ for all } m \in \mathcal{N} \setminus \{n\}. \tag{46}$$

Hence, at time t_1, all EVs are fully allocated with respect to $(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})_{t_1}$. Similarly, with respect to $(\widetilde{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})_{t_1}$,

$$\widetilde{x}_{nt_1} = \widetilde{d}_{nt_1}, \qquad \widetilde{x}_{mt_1} = d^*_{mt_1} \text{ for all } m \in \mathcal{N} \setminus \{n\}. \tag{47}$$

By (46) and (47), the difference in the payments of the n-th EV at time t_1 with respect to $(\widetilde{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})_{t_1}$ and $(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})_{t_1}$ is given by,

$$\begin{aligned} \Delta\tau_{nt_1} &\triangleq \tau_{nt_1}\Big((\widetilde{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})_{t_1}\Big) - \tau_{nt_1}\Big((\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})_{t_1}\Big) \\ &= c_t(D_{t_1} + \sum_{m\neq n} d^*_{mt_1} + \widetilde{d}_{nt_1}) \\ &\qquad - c_t(D_{t_1} + \sum_{m\neq n} d^*_{mt_1} + \widehat{d}_{nt_1}). \end{aligned} \tag{48}$$

For the n-th EV at time t_2, the difference in payments with respect to $(\widetilde{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})_{t_2}$ and $(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})_{t_2}$ is given by,

$$\begin{aligned} \Delta\tau_{nt_2} &\triangleq \tau_{nt_2}\Big((\widetilde{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})_{t_2}\Big) - \tau_{nt_2}\Big((\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})_{t_2}\Big) \\ &= c_t(D_{t_2} + \sum_{m\neq n} \widetilde{x}_{mt_2} + \widetilde{x}_{nt_2}) \\ &\qquad - c_t(D_{t_2} + \sum_{m\neq n} \widehat{x}_{mt_2} + \widehat{x}_{nt_2}) \\ &\qquad + \sum_{m\neq n} \beta^*_{mt_2}(\widehat{x}_{mt_2} - \widetilde{x}_{mt_2}). \end{aligned} \tag{49}$$

The last term of (49) can be simplified by recalling from Lemma 3.2 that all EVs, $k \in \mathcal{N}$, with $d^*_{kt_2} > 0$ share the same value for $\beta^*_{kt_2}$. Denoting that common value by $\beta^*_{\diamond t_2}$ allows the last term to be expressed as $\beta^*_{\diamond t_2} \sum_{m\neq n}(\widehat{x}_{mt_2} - \widetilde{x}_{mt_2})$.

It follows from (27d) that for the n-th EV, the difference in payments at times $t \neq t_1, t_2$, with respect to $(\widetilde{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})_t$ and $(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})_t$ is,

$$\Delta\tau_{nt} \triangleq \tau_{nt}\big((\widetilde{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})_t\big) - \tau_{nt}\big((\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})_t\big) = 0, \; \forall t \neq t_1, t_2. \tag{50}$$

Thus, by (48)-(50), the difference in the payments of the n-th EV with respect to $(\widetilde{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})$ and $(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})$ satisfies,

$$\Delta\tau_n \triangleq \tau_n(\widetilde{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n}) - \tau_n(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n}) = \Delta\tau_{nt_1} + \Delta\tau_{nt_2}. \tag{51}$$

The difference in utility of the n-th EV, with respect to $(\widetilde{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})$ and $(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})$, is given by,

$$\begin{aligned} \Delta w_n &\triangleq w_n(\widetilde{\boldsymbol{x}}_n) - w_n(\widehat{\boldsymbol{x}}_n) \\ &= -\delta_n\Big(\sum_{t\in\mathcal{T}} \widetilde{x}_{nt} - \Gamma_n\Big)^2 + \delta_n\Big(\sum_{t\in\mathcal{T}} \widehat{x}_{nt} - \Gamma_n\Big)^2 \\ &\qquad + f_n(\widehat{d}_{nt_1}) - f_n(\widetilde{d}_{nt_1}) + f_n(\widehat{x}_{nt_2}) - f_n(\widetilde{x}_{nt_2}). \end{aligned} \tag{52}$$

By (51) and (52), the difference in the payoff of the n-th EV, subject to $(\widetilde{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})$ and $(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})$, becomes,

$$\Delta u_n \triangleq u_n(\widetilde{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n}) - u_n(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n}) = \Delta w_n - \Delta\tau_n. \tag{53}$$

To establish (28), firstly the case with $\widehat{d}_{nt_2} = d^*_{nt_2}$ will be addressed, then the three cases $\widehat{d}_{nt_2}, \widetilde{d}_{nt_2} \in \mathcal{R}_i, i = 1, 2, 3$ will be considered separately.

Case I, $\widehat{d}^*_{nt_2} < \widetilde{d}_{nt_2} < \widehat{d}_{nt_2} = d^*_{nt_2}$

Because $\widehat{d}_{nt_2} = d^*_{nt_2}$,

$$\widehat{\beta}_{nt_2} = \beta_{nt_2}(d^*_{nt_2}, A) > \beta_{nt_2}(d^*_{nt_2}, \sum_{t\in\mathcal{T}} d^*_{nt}) = \beta^*_{nt_2}.$$

Likewise, with $\widehat{d}^*_{nt_2} < \widetilde{d}_{nt_2} < \widehat{d}_{nt_2} = d^*_{nt_2}$,

$$\widetilde{\beta}_{nt_2} = \beta_{nt_2}(\widetilde{d}_{nt_2}, A) > \beta_{nt_2}(\widehat{d}^2_{nt_2}, A) = \beta^*_{nt_2}.$$

A similar argument to that used to establish (46),(47) for t_1 shows that all EVs are fully allocated at t_2 with respect to both $(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})_{t_2}$ and $(\widetilde{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})_{t_2}$:

$$\begin{aligned} &\widehat{x}_{nt_2} = \widehat{d}_{nt_2} = d^*_{nt_2}, \qquad \widehat{x}_{mt_2} = d^*_{mt_2} \text{ for all } m \in \mathcal{N} \setminus \{n\}, \\ &\widetilde{x}_{nt_2} = \widetilde{d}_{nt_2}, \qquad \widetilde{x}_{mt_2} = d^*_{mt_2} \text{ for all } m \in \mathcal{N} \setminus \{n\}. \end{aligned}$$

Substituting these allocations into (53) gives,

$$\begin{aligned} \Delta u_n &= f_n(\widehat{d}_{nt_1}) - f_n(\widetilde{d}_{nt_1}) + f_n(\widehat{d}_{nt_2}) - f_n(\widetilde{d}_{nt_2}) \\ &\qquad - \Big(c_t(D_{t_1} + \sum_{m\neq n} d^*_{mt_1} + \widetilde{d}_{nt_1}) \\ &\qquad\quad - c_t(D_{t_1} + \sum_{m\neq n} d^*_{mt_1} + \widehat{d}_{nt_1}) \\ &\qquad\quad + c_t(D_{t_2} + \sum_{m\neq n} d^*_{mt_2} + \widetilde{d}_{nt_2}) \\ &\qquad\quad - c_t(D_{t_2} + \sum_{m\neq n} d^*_{mt_2} + \widehat{d}_{nt_2})\Big) \\ &= g_{nt_1}(\widehat{d}_{nt_1}) - g_{nt_1}(\widetilde{d}_{nt_1}) + g_{nt_2}(\widehat{d}_{nt_2}) - g_{nt_2}(\widetilde{d}_{nt_2}) \\ &> g'_{nt_1}(\widehat{d}^*_{nt_1})\big(\widehat{d}_{nt_1} - \widetilde{d}_{nt_1}\big) + g'_{nt_2}(\widehat{d}^*_{nt_2})\big(\widehat{d}_{nt_2} - \widetilde{d}_{nt_2}\big) \\ &= \mu(\widehat{d}_{nt_1} - \widetilde{d}_{nt_1} + \widehat{d}_{nt_2} - \widetilde{d}_{nt_2}) \\ &= 0, \end{aligned}$$

where the inequality holds due to the convexity of $g_{nt}(\cdot)$ and the subsequent equality follows from (26). Therefore, (28) is satisfied in this case.

Case II, $\widehat{d}_{nt_2}, \widetilde{d}_{nt_2} \in \mathcal{R}_1$

The initial step in showing (28) is to determine the allocations of all EVs at time t_2 with respect to the bid profile $(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})_{t_2}$. Firstly, consider $\widehat{d}_{nt_2} \in \text{Int}(\mathcal{R}_1)$. Then,

$$\begin{aligned} \widehat{\beta}_{nt_2} > \widehat{\beta}^1_{nt_2} &= c'_t(D_{t_2} + \sum_{m\neq n} d^*_{mt_2} + \widehat{d}^1_{nt_2}) \\ &> c'_t(D_{t_2} + \sum_{m\neq n} d^*_{mt_2} + \widehat{d}_{nt_2}), \end{aligned}$$

so it follows from the KKT conditions (8) that $\widehat{x}_{nt_2} = \widehat{d}_{nt_2}$.

Now consider the case with $\widehat{d}_{nt_2} = \widehat{d}^1_{nt_2}$, the upper boundary of $\mathcal{R}_1$. In this case, $\widehat{\beta}_{nt_2} = c'_t(D_{t_2} + \sum_{m\neq n} d^*_{mt_2} + \widehat{d}_{nt_2})$. Assume $\widehat{x}_{nt_2} < \widehat{d}_{nt_2}$. Then due to the convexity of $c_t(\cdot)$,

$$c'_t(D_{t_2} + \sum_{m\neq n} d^*_{mt_2} + \widehat{x}_{nt_2}) < c'_t(D_{t_2} + \sum_{m\neq n} d^*_{mt_2} + \widehat{d}_{nt_2}) = \widehat{\beta}_{nt_2}.$$

But (8) then implies $\widehat{\sigma}_{nt_2} > 0$ and therefore that $\widehat{x}_{nt_2} = \widehat{d}_{nt_2}$. Hence a contradiction, so $\widehat{x}_{nt_2} = \widehat{d}_{nt_2}$.

If $\widehat{d}_{nt_2} \geq \sum_{k\in\mathcal{N}} d^*_{kt_2}$ then it can be shown by contradiction that $\sum_{m\neq n} \widehat{x}_{mt_2} = 0$. Assuming $\sum_{m\neq n} \widehat{x}_{mt_2} > 0$ gives,

$$c'_t(D_{t_2} + \sum_{k\neq n} \widehat{x}_{kt_2} + \widehat{x}_{nt_2}) > c'_t(D_{t_2} + \sum_{k\in\mathcal{N}} d^*_{kt_2}) \geq \beta^*_{mt_2},$$

for all $m \in \mathcal{N} \setminus \{n\}$. But (8) then implies $\widehat{x}_{mt_2} = 0$ for all $m \in \mathcal{N} \setminus \{n\}$, hence a contradiction. Alternatively, if $\widehat{d}_{nt_2} < \sum_{k\in\mathcal{N}} d^*_{kt_2}$ then it can be shown, once again by contradiction, that $\sum_{k\in\mathcal{N}} \widehat{x}_{kt_2} = \sum_{k\in\mathcal{N}} d^*_{kt_2}$. Consider $\sum_{k\in\mathcal{N}} \widehat{x}_{kt_2} > \sum_{k\in\mathcal{N}} d^*_{kt_2}$. Then $c'_t(D_{t_2} + \sum_{k\in\mathcal{N}} \widehat{x}_{kt_2}) > \beta^*_{mt_2}$ for $m \in \mathcal{N}\setminus\{n\}$, with (8) implying $\widehat{x}_{mt_2} = 0$, hence a contradiction. If $\sum_{k\in\mathcal{N}} \widehat{x}_{kt_2} < \sum_{k\in\mathcal{N}} d^*_{kt_2}$, then $c'_t(D_{t_2} + \sum_{k\in\mathcal{N}} \widehat{x}_{kt_2}) < \beta^*_{mt_2}$, with (8) implying $\widehat{x}_{mt_2} = d^*_{mt_2}$. This leads to another contradiction, as $\sum_{k\in\mathcal{N}} \widehat{x}_{kt_2} = \sum_{m\neq n} d^*_{mt_2} + \widehat{d}_{nt_2} > \sum_{k\in\mathcal{N}} d^*_{kt_2}$. Summarizing,

$$\begin{aligned}
&\widehat{x}_{nt_2} = \widehat{d}_{nt_2}, \ \sum_{m\neq n} \widehat{x}_{mt_2} > 0, \ \sum_{m\neq n} \widehat{x}_{mt_2} + \widehat{d}_{nt_2} = \sum_{k\in\mathcal{N}} d^*_{kt_2}, \\
&\qquad \text{if } \widehat{d}_{nt_2} < \sum_{k\in\mathcal{N}} d^*_{kt_2}, \qquad (54a)\\
&\widehat{x}_{nt_2} = \widehat{d}_{nt_2}, \ \sum_{m\neq n} \widehat{x}_{mt_2} = 0, \ \sum_{m\neq n} \widehat{x}_{mt_2} + \widehat{d}_{nt_2} \geq \sum_{k\in\mathcal{N}} d^*_{kt_2}, \\
&\qquad \text{if } \widehat{d}_{nt_2} \geq \sum_{k\in\mathcal{N}} d^*_{kt_2}. \qquad (54b)
\end{aligned}$$

Similarly, the above analysis also holds for the bid profile $(\widetilde{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})_{t_2}$ so,

$$\begin{aligned}
&\widetilde{x}_{nt_2} = \widetilde{d}_{nt_2}, \ \sum_{m\neq n} \widetilde{x}_{mt_2} > 0, \ \sum_{m\neq n} \widetilde{x}_{mt_2} + \widetilde{d}_{nt_2} = \sum_{k\in\mathcal{N}} d^*_{kt_2}, \\
&\qquad \text{if } \widetilde{d}_{nt_2} < \sum_{k\in\mathcal{N}} d^*_{kt_2}, \qquad (55a)\\
&\widetilde{x}_{nt_2} = \widetilde{d}_{nt_2}, \ \sum_{m\neq n} \widetilde{x}_{mt_2} = 0, \ \sum_{m\neq n} \widetilde{x}_{mt_2} + \widetilde{d}_{nt_2} \geq \sum_{k\in\mathcal{N}} d^*_{kt_2}, \\
&\qquad \text{if } \widetilde{d}_{nt_2} \geq \sum_{k\in\mathcal{N}} d^*_{kt_2}. \qquad (55b)
\end{aligned}$$

Substituting into (51) gives,

$$\begin{aligned}
\Delta\tau_n &= c_t(D_{t_1} + \sum_{m\neq n} d^*_{mt_1} + \widetilde{d}_{nt_1}) - c_t(D_{t_1} + \sum_{m\neq n} d^*_{mt_1} + \widehat{d}_{nt_1}) \\
&\quad + c_t(D_{t_2} + \sum_{m\neq n} \widetilde{x}_{mt_2} + \widetilde{d}_{nt_2}) + \beta^*_{\diamond t_2} \sum_{m\neq n} (\widehat{x}_{mt_2} - \widetilde{x}_{mt_2}) \\
&\quad - c_t(D_{t_2} + \sum_{m\neq n} \widehat{x}_{mt_2} + \widehat{d}_{nt_2}).
\end{aligned}$$

Because $\widehat{x}_{nt_1} + \widehat{x}_{nt_2} = \widetilde{x}_{nt_1} + \widetilde{x}_{nt_2}$ and $\widehat{x}_{nt} = \widetilde{x}_{nt}$ for all $t \neq t_1, t_2$, it follows that $\sum_t \widehat{x}_{nt} = \sum_t \widetilde{x}_{nt}$, and so (52) becomes,

$$\Delta w_n = f_n(\widehat{d}_{nt_1}) + f_n(\widehat{d}_{nt_2}) - f_n(\widetilde{d}_{nt_1}) - f_n(\widetilde{d}_{nt_2}).$$

Three subcases must be considered, depending on the relative values of $\widetilde{d}_{nt_2}$, $\widehat{d}_{nt_2}$ and $\sum_{k\in\mathcal{N}} d^*_{kt_2}$.

Case II.1, $\widetilde{d}_{nt_2} < \widehat{d}_{nt_2} < \sum_{k=1}^N d^*_{kt_2}$: In this case, Δu_n defined in (53) is established using (54a) and (55a),

$$\begin{aligned}
\Delta u_n &= g_{nt_1}(\widehat{d}_{nt_1}) - g_{nt_1}(\widetilde{d}_{nt_1}) + f_n(\widehat{d}_{nt_2}) - f_n(\widetilde{d}_{nt_2}) \\
&\quad - \beta^*_{\diamond t_2}(\widetilde{d}_{nt_2} - \widehat{d}_{nt_2}) \qquad (56a)\\
&> g'_{nt_1}(\widehat{d}^*_{nt_1})(\widehat{d}_{nt_1} - \widetilde{d}_{nt_1}) \\
&\quad + \big(f'_n(d^*_{nt_2}) + \beta^*_{\diamond t_2}\big)(\widehat{d}_{nt_2} - \widetilde{d}_{nt_2}) \qquad (56b)\\
&= g'_{nt_1}(\widehat{d}^*_{nt_1})(\widehat{d}_{nt_1} - \widetilde{d}_{nt_1}) + g'_{nt_2}(d^*_{nt_2})(\widehat{d}_{nt_2} - \widetilde{d}_{nt_2}), \qquad (56c)
\end{aligned}$$

where (56a) holds by the specification of $g_{nt}(\cdot)$ given in (25) and substitution from (54a) and (55a); (56b) holds by the convexity of $g_{nt}(\cdot)$ together with (27a), and the convexity of $f_n(\cdot)$ together with (29); and (56c) holds by (10) in Lemma 3.2 and (25).

From (22), $\widehat{d}^*_{nt_2} < d^*_{nt_2}$, so $g'_{nt_2}(d^*_{nt_2}) > g'_{nt_2}(\widehat{d}^*_{nt_2})$ due to the convexity of $g_{nt_2}(\cdot)$. By construction, $\widehat{d}^*_{nt_1} > 0$, so (26) gives $g'_{nt_2}(\widehat{d}^*_{nt_2}) \geq g'_{nt_1}(\widehat{d}^*_{nt_1}) = \mu$. Therefore, because (29) ensures $\widehat{d}_{nt_2} > \widetilde{d}_{nt_2}$, (56c) gives,

$$\Delta u_n > \mu(\widehat{d}_{nt_1} - \widetilde{d}_{nt_1} + \widehat{d}_{nt_2} - \widetilde{d}_{nt_2}) = 0, \qquad (57)$$

where the final equality holds by (27c).

Case II.2, $\widetilde{d}_{nt_2} < \sum_{k\in\mathcal{N}} d^*_{kt_2} \leq \widehat{d}_{nt_2}$: In this case, Δu_n is governed by (54b) and (55a), giving,

$$\begin{aligned}
\Delta u_n &= g_{nt_1}(\widehat{d}_{nt_1}) - g_{nt_1}(\widetilde{d}_{nt_1}) + f_n(\widehat{d}_{nt_2}) - f_n(\widetilde{d}_{nt_2}) \\
&\quad - c_t(D_{t_2} + \sum_{m\neq n} \widetilde{x}_{mt_2} + \widetilde{d}_{nt_2}) + c_t(D_{t_2} + \widehat{d}_{nt_2}) \\
&\quad + \beta^*_{\diamond t_2} \sum_{m\neq n} \widetilde{x}_{mt_2} \qquad (58a)\\
&> g'_{nt_1}(\widehat{d}^*_{nt_1})(\widehat{d}_{nt_1} - \widetilde{d}_{nt_1}) + f'_n(d^*_{nt_2})(\widehat{d}_{nt_2} - \widetilde{d}_{nt_2}) \\
&\quad + c'_t(D_{t_2} + \sum_{k\in\mathcal{N}} d^*_{kt_2})(\widehat{d}_{nt_2} - \sum_{m\neq n} \widetilde{x}_{mt_2} - \widetilde{d}_{nt_2}) \\
&\quad + \beta^*_{\diamond t_2} \sum_{m\neq n} \widetilde{x}_{mt_2} \qquad (58b)\\
&= g'_{nt_1}(\widehat{d}^*_{nt_1})(\widehat{d}_{nt_1} - \widetilde{d}_{nt_1}) + f'_n(d^*_{nt_2})(\widehat{d}_{nt_2} - \widetilde{d}_{nt_2}) \\
&\quad + \beta^*_{\diamond t_2}(\widehat{d}_{nt_2} - \widetilde{d}_{nt_2}), \qquad (58c)
\end{aligned}$$

where (58a) holds by (53) and (25); (58b) holds by the convexity of $g_{nt}(\cdot)$ together with (27a), the convexity of $f_n(\cdot)$ together with (29), and the convexity of $c_t(\cdot)$ using (55a); and (58c) holds by (10). Proceeding as in (56c),(57) yields $\Delta u_n > 0$.

Case II.3, $\sum_{k\in\mathcal{N}} d^*_{kt_2} \leq \widetilde{d}_{nt_2} < \widehat{d}_{nt_2}$: In this case, Δu_n uses (54b) and (55b) to give,

$$\begin{aligned}
\Delta u_n &= g_{nt_1}(\widehat{d}_{nt_1}) - g_{nt_1}(\widetilde{d}_{nt_1}) + f_n(\widehat{d}_{nt_2}) - f_n(\widetilde{d}_{nt_2}) \\
&\quad - c_t(D_{t_2} + \widetilde{d}_{nt_2}) + c_t(D_{t_2} + \widehat{d}_{nt_2}) \qquad (59a)\\
&> g'_{nt_1}(\widehat{d}^*_{nt_1})(\widehat{d}_{nt_1} - \widetilde{d}_{nt_1}) + f'_n(d^*_{nt_2})(\widehat{d}_{nt_2} - \widetilde{d}_{nt_2}) \\
&\quad + c'_t(D_{t_2} + \widetilde{d}_{nt_2})(\widehat{d}_{nt_2} - \widetilde{d}_{nt_2}) \qquad (59b)\\
&\geq g'_{nt_1}(\widehat{d}^*_{nt_1})(\widehat{d}_{nt_1} - \widetilde{d}_{nt_1}) \\
&\quad + \Big(f'_n(d^*_{nt_2}) + c'_t(D_{t_2} + \sum_{m\neq n} d^*_{mt_2} + d^*_{nt_2})\Big) \\
&\quad \times (\widehat{d}_{nt_2} - \widetilde{d}_{nt_2}) \qquad (59c)\\
&= g'_{nt_1}(\widehat{d}^*_{nt_1})(\widehat{d}_{nt_1} - \widetilde{d}_{nt_1}) + g'_{nt_2}(d^*_{nt_2})(\widehat{d}_{nt_2} - \widetilde{d}_{nt_2}), \qquad (59d)
\end{aligned}$$

where (59a) holds by (53) and (25); (59b) holds by the convexity of $g_{nt}(\cdot)$ together with (27a), and the convexity of $f_n(\cdot)$ and $c_t(\cdot)$ together with (29); (59c) holds by the convexity of $c_t(\cdot)$ with $\widetilde{d}_{nt_2} \geq \sum_{k\in\mathcal{N}} d^*_{kt_2}$; and (59d) holds by (25). Proceeding as in (57) yields $\Delta u_n > 0$.

Hence, $\Delta u_n > 0$ whenever $\widehat{d}_{nt_2}, \widetilde{d}_{nt_2} \in \mathcal{R}_1$.

Case III, $\widehat{d}_{nt_2}, \widetilde{d}_{nt_2} \in$ Int$(\mathcal{R}_2)$

The situation where $\widehat{d}_{nt_2} \in \mathcal{R}_2$ will be considered as two separate cases. Case III, presented here, discusses $\widehat{d}_{nt_2} \in$ Int$(\mathcal{R}_2)$, while Case IV addresses $\widehat{d}_{nt_2} = \widehat{d}^2_{nt_2}$, the upper boundary of $\mathcal{R}_2$.

Consider the allocations of all EVs at time t_2 with respect to the bid profile $(\widehat{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})_{t_2}$. If $\widehat{d}_{nt_2} < \sum_{k\in\mathcal{N}} d^*_{kt_2}$, then the argument presented in Case II can again be used to show that $\sum_{k\in\mathcal{N}} \widehat{x}_{kt_2} = \sum_{k\in\mathcal{N}} d^*_{kt_2}$. Also, because $\widehat{\beta}_{nt_2} > \beta^*_{nt_2} = c'_t(D_{t_2} + \sum_{k\in\mathcal{N}} d^*_{kt_2})$, (8) implies $\widehat{x}_{nt_2} = \widehat{d}_{nt_2}$. Similar outcomes hold for the bid profile

$(\widetilde{\boldsymbol{b}}_n, \boldsymbol{b}^*_{-n})_{t_2}$ as $\widetilde{d}_{nt_2} < \widehat{d}_{nt_2}$. Therefore, (54a) and (55a) are again applicable.

However, if $\widehat{d}_{nt_2} > \sum_{k\in\mathcal{N}} d^*_{kt_2}$, then because $\widehat{\beta}_{nt_2} < \widehat{\beta}^1_{nt_2} = c'_t(D_t + \sum_{m\neq n} d^*_{mt_2} + \widehat{d}^1_{nt_2})$, there is no guarantee that $\widehat{x}_{nt_2} = \widehat{d}_{nt_2}$. Whether or not (54b) holds depends on the comparison between $\widehat{\beta}_{nt_2}$ and $c'_t(D_{t_2} + \sum_{m\neq n} \widehat{x}_{mt_2} + \widehat{d}_{nt_2})$. Similarly, for the bid profile $(\widetilde{b}_{nt_2}, \boldsymbol{b}^*_{-n,t_2})$, there is no guarantee that (55b) holds.

Three subcases must be considered for $\widehat{\boldsymbol{x}}_{t_2}$ and $\widetilde{\boldsymbol{x}}_{t_2}$, depending on the relative values of $\widetilde{d}_{nt_2}$, $\widehat{d}_{nt_2}$ and $\sum_{k\in\mathcal{N}} d^*_{kt_2}$.

Case III.1, $\widetilde{d}_{nt_2} < \widehat{d}_{nt_2} < \sum_{k\in\mathcal{N}} d^*_{kt_2}$: Analysis of Δu_n in this case is identical to that of Case II.1, so $\Delta u_n > 0$.

Case III.2, $\widetilde{d}_{nt_2} < \sum_{k\in\mathcal{N}} d^*_{kt_2} \le \widehat{d}_{nt_2}$: Because $\widehat{\beta}_{nt_2} > \beta^*_{nt_2}$, satisfying (8) for the n-th EV results in $\sum_{k\in\mathcal{N}} \widehat{x}_{kt_2} \ge \sum_{k\in\mathcal{N}} d^*_{kt_2}$, with equality holding only if $\widehat{x}_{nt_2} = \widehat{d}_{nt_2} = \sum_{k\in\mathcal{N}} d^*_{kt_2}$ and $\sum_{m\neq n} \widehat{x}_{mt_2} = 0$. If the inequality is strict, then $c'_t(D_{t_2} + \sum_{k\in\mathcal{N}} \widehat{x}_{kt_2}) > \beta^*_{mt_2}$ for all $m \in \mathcal{N}\setminus\{n\}$, with (8) implying $\widehat{x}_{mt_2} = 0$. Hence,

$$\sum_{k\in\mathcal{N}} d^*_{kt_2} \le \widehat{x}_{nt_2} \le \widehat{d}_{nt_2}, \qquad \sum_{m\neq n} \widehat{x}_{mt_2} = 0. \tag{60}$$

The applicability of (54b) reverts to a comparison between $\widehat{\beta}_{nt_2}$ and $c'_t(D_{t_2} + \widehat{d}_{nt_2})$:

- If $\widehat{\beta}_{nt_2} \ge c'_t(D_{t_2} + \widehat{d}_{nt_2})$ then it can be verified that (54b) holds. Thus, $\Delta u_n > 0$, since the analysis in this case is identical to that developed in Case II.2.
- If $\widehat{\beta}_{nt_2} < c'_t(D_{t_2} + \widehat{d}_{nt_2})$, (54b) does not hold. Rather, Δu_n can be established using (53), (25), (55a) and (60),

$$\begin{aligned}
\Delta u_n &= g_{nt_1}(\widehat{d}_{nt_1}) - g_{nt_1}(\widetilde{d}_{nt_1}) - \delta_n\Big(\sum_{t\in\mathcal{T}} \widetilde{x}_{nt} - \Gamma_n\Big)^2 \\
&\quad + \delta_n\Big(\sum_{t\in\mathcal{T}} \widehat{x}_{nt} - \Gamma_n\Big)^2 + f_n(\widehat{x}_{nt_2}) - f_n(\widetilde{d}_{nt_2}) \\
&\quad - c_t(D_{t_2} + \sum_{m\neq n} \widetilde{x}_{mt_2} + \widetilde{d}_{nt_2}) + c_t(D_{t_2} + \widehat{x}_{nt_2}) \\
&\quad + \beta^*_{\diamond t_2} \sum_{m\neq n} \widetilde{x}_{mt_2} \\
&> g'_{nt_1}(\widehat{d}^*_{nt_1})(\widehat{d}_{nt_1} - \widetilde{d}_{nt_1}) + f'_n(d^*_{nt_2})(\widehat{x}_{nt_2} - \widetilde{d}_{nt_2}) \\
&\quad + g'_{nt_2}(d^*_{nt_2})(\widetilde{d}_{nt_1} - \widehat{d}_{nt_1} + \widetilde{d}_{nt_2} - \widehat{x}_{nt_2}) \\
&\quad + \beta^*_{\diamond t_2} \sum_{m\neq n} \widetilde{x}_{mt_2} + c'_t(D_{t_2} + \sum_{m\neq n} d^*_{mt_2} + d^*_{nt_2}) \\
&\qquad \times \big(\widehat{x}_{nt_2} - \sum_{m\neq n} \widetilde{x}_{mt_2} - \widetilde{d}_{nt_2}\big) && \text{(61a)} \\
&= g'_{nt_1}(\widehat{d}^*_{nt_1})(\widehat{d}_{nt_1} - \widetilde{d}_{nt_1}) + g'_{nt_2}(d^*_{nt_2})(\widetilde{d}_{nt_1} - \widehat{d}_{nt_1}) && \text{(61b)} \\
&> \mu(\widehat{d}_{nt_1} - \widetilde{d}_{nt_1}) + \mu(\widetilde{d}_{nt_1} - \widehat{d}_{nt_1}) && \text{(61c)} \\
&= 0,
\end{aligned}$$

where (61a) holds by the convexity of $g_{nt}(\cdot)$ together with (27a), the convexity of $f_n(\cdot)$ together with (29) and (60), the convexity of $c_t(\cdot)$ together with (55a) and (60), and the concavity of $-\delta_n(\sum_{t\in\mathcal{T}} x_{nt} - \Gamma_n)^2$ together with Lemma A.1 specified below, recalling that $\widehat{\boldsymbol{b}}_n, \widetilde{\boldsymbol{b}}_n \in \mathcal{B}_n(A)$ with $\sum_{t\in\mathcal{T}} d_{nt} = A < \sum_{t\in\mathcal{T}} d^*_{nt}$, and that $\sum_{t\in\mathcal{T}} \widetilde{x}_{nt} - \sum_{t\in\mathcal{T}} \widehat{x}_{nt} = \widetilde{d}_{nt_1} + \widetilde{d}_{nt_2} - (\widehat{d}_{nt_1} + \widehat{x}_{nt_2}) \ge 0$; (61b) holds by (10) in Lemma 3.2 together with (25); and (61c) follows the same justification as (57) though using (27a).

Lemma A.1: Consider an allocation $\boldsymbol{x}_n(\boldsymbol{b}) \equiv (x_{nt}, t\in\mathcal{T})$ with respect to a bid profile $\boldsymbol{b}$, such that $\sum_{t\in\mathcal{T}} d_{nt} < \sum_{t\in\mathcal{T}} d^*_{nt}$. Then:

$$\frac{\partial}{\partial x_{nt}}\Big(-\delta_n\Big(\sum_{t\in\mathcal{T}} x_{nt} - \Gamma_n\Big)^2\Big) > g'_{nt}(d^*_{nt}) > \mu, \tag{62}$$

for all $t\in\mathcal{T}$, where g_{nt} is defined in Lemma 4.3.

Proof of Lemma A.1.

$$\begin{aligned}
\frac{\partial}{\partial x_{nt}}\Big(-\delta_n\Big(\sum_{t\in\mathcal{T}} x_{nt} - \Gamma_n\Big)^2\Big) &= 2\delta_n\Big(\Gamma_n - \sum_{t\in\mathcal{T}} x_{nt}\Big) \\
&> 2\delta_n\Big(\Gamma_n - \sum_{t\in\mathcal{T}} d^*_{nt}\Big) && \text{(63a)} \\
&= \beta^*_{nt} + f'_n(d^*_{nt}) && \text{(63b)} \\
&= c'_t(D_t + \sum_{m\neq n} d^*_{mt} + d^*_{nt}) + f'_n(d^*_{nt}) && \text{(63c)} \\
&= g'_{nt}(d^*_{nt}) && \text{(63d)} \\
&> \mu, && \text{(63e)}
\end{aligned}$$

where (63a) holds because $\sum_{t\in\mathcal{T}} x_{nt} \le \sum_{t\in\mathcal{T}} d_{nt} < \sum_{t\in\mathcal{T}} d^*_{nt}$; (63b) follows from $\beta^*_{nt} = \frac{\partial}{\partial d_{nt}} w_n(\boldsymbol{d}^*_n) = -f'_n(d^*_{nt}) + 2\delta_n(\Gamma_n - \sum_{t\in\mathcal{T}} d^*_{nt})$; (63c) holds by (10) in Lemma 3.2; (63d) holds by the specification of g_{nt} in (25); and (63e) holds by Lemma 4.3 and the convexity of g_{nt}.

End of proof of Lemma A.1.

Case III.3, $\sum_{k\in\mathcal{N}} d^*_{kt_2} \le \widetilde{d}_{nt_2} < \widehat{d}_{nt_2}$: Using the same argument as in Case III.2 gives $\sum_{m\neq n} \widehat{x}_{mt_2} = \sum_{m\neq n} \widetilde{x}_{mt_2} = 0$. Analysis of Δu_n depends on the relative values of $\widetilde{d}_{nt_2}$, $\widehat{x}_{nt_2}$ and $\widehat{d}_{nt_2}$, keeping in mind from Lemma 4.1 that $\beta_{nt_2}(\widetilde{d}_{nt_2}, A) > \beta_{nt_2}(\widehat{d}_{nt_2}, A)$.

- If $\widehat{x}_{nt_2} = \widehat{d}_{nt_2}$ then $\widetilde{x}_{nt_2} = \widetilde{d}_{nt_2}$ must also hold. Analysis of Δu_n in this case is identical to that developed in Case II.3.
- If $\widetilde{d}_{nt_2} \le \widehat{x}_{nt_2} < \widehat{d}_{nt_2}$ then $\widetilde{x}_{nt_2} = \widetilde{d}_{nt_2} \le \widehat{x}_{nt_2}$. Analysis of Δu_n follows that of Case III.2.
- If $\widehat{x}_{nt_2} < \widetilde{d}_{nt_2} < \widehat{d}_{nt_2}$ then $\widehat{x}_{nt_2} < \widetilde{x}_{nt_2}$, and Δu_n satisfies,

$$\begin{aligned}
\Delta u_n &> g'_{nt_1}(\widehat{d}^*_{nt_1})(\widehat{d}_{nt_1} - \widetilde{d}_{nt_1}) + f'_n(\widetilde{x}_{nt_2})(\widehat{x}_{nt_2} - \widetilde{x}_{nt_2}) \\
&\quad + c'_t(D_{t_2} + \widetilde{x}_{nt_2})(\widetilde{x}_{nt_2} - \widehat{x}_{nt_2}) \\
&\quad + g'_{nt_2}(d^*_{nt_2})(\widetilde{d}_{nt_1} - \widehat{d}_{nt_1}) \\
&\quad + 2\delta_n\Big(\Gamma_n - \sum_{t\in\mathcal{T}} \widetilde{x}_{nt}\Big)(\widetilde{x}_{nt_2} - \widehat{x}_{nt_2}) && \text{(64)} \\
&> f'_n(\widetilde{x}_{nt_2})(\widehat{x}_{nt_2} - \widetilde{x}_{nt_2}) \\
&\quad + c'_t(D_{t_2} + \widetilde{x}_{nt_2})(\widehat{x}_{nt_2} - \widetilde{x}_{nt_2}) \\
&\quad + 2\delta_n\Big(\Gamma_n - \sum_{t\in\mathcal{T}} \widetilde{x}_{nt}\Big)(\widetilde{x}_{nt_2} - \widehat{x}_{nt_2}), && \text{(65)}
\end{aligned}$$

where (64) holds by the convexity of $g_{nt_1}(\cdot)$ together with (27a), the convexity of $f_n(\cdot)$ and $c_t(\cdot)$, the concavity of $-\delta_n(\Gamma_n - \sum_{t\in\mathcal{T}} x_{nt})^2$ and Lemma A.1; and (65) makes use of (61b). Further analysis uses $\widetilde{x}_{nt} \le \widetilde{d}_{nt}$ for all $t\in\mathcal{T}$ to give,

$$\begin{aligned}
2\delta_n\Big(\Gamma_n - \sum_{t\in\mathcal{T}} \widetilde{x}_{nt}\Big) &\ge 2\delta_n\Big(\Gamma_n - \sum_{t\in\mathcal{T}} \widetilde{d}_{nt}\Big) \\
&= f'_n(\widetilde{d}_{nt_2}) + \widetilde{\beta}_{nt_2}, && \text{(66)}
\end{aligned}$$

where the equality follows from (11). Because $\widetilde{x}_{nt_2} > 0$ and $\sum_{m\neq n} \widetilde{x}_{mt_2} = 0$, (8) gives $\widetilde{\beta}_{nt_2} \ge c'_t(D_{t_2} + \widetilde{x}_{nt_2})$. Therefore,

$$\begin{aligned}
2\delta_n\Big(\Gamma_n - \sum_{t\in\mathcal{T}} \widetilde{d}_{nt}\Big) &\ge f'_n(\widetilde{d}_{nt_2}) + c'_t(D_{t_2} + \widetilde{x}_{nt_2}) \\
&\ge f'_n(\widetilde{x}_{nt_2}) + c'_t(D_{t_2} + \widetilde{x}_{nt_2}). && \text{(67)}
\end{aligned}$$

Because $\widehat{x}_{nt_2} < \widetilde{x}_{nt_2}$, (65) and (67) ensure $\Delta u_n > 0$.

Hence, $\Delta u_n > 0$ whenever $\widehat{d}_{nt_2}, \widetilde{d}_{nt_2} \in \mathrm{Int}(\mathcal{R}_2)$.

Case IV, $\widehat{d}_{nt_2} = \widehat{d}^2_{nt_2}, \widetilde{d}_{nt_2} \in \mathcal{R}_2$

In this case, $\widehat{\beta}_{nt_2} = \beta^*_{nt_2}$, so (8) ensures that $\sum_{k\in\mathcal{N}} \widehat{x}_{kt_2} = \sum_{k\in\mathcal{N}} d^*_{kt_2}$ and $d^*_{nt_2} \le \widehat{x}_{nt_2} \le \widehat{d}^2_{nt_2}$.

Case IV.1, $\widetilde{d}_{nt_2} < \sum_{k\in\mathcal{N}} d^*_{kt_2}$*:* Using the same argument as in Case III, (55a) is again applicable.

If $\widehat{x}_{nt_2} > \widetilde{x}_{nt_2} = \widetilde{d}_{nt_2}$, Δu_n can be established by,

$$\begin{aligned}\Delta u_n &> g'_{nt_1}(\widehat{d}^*_{nt_1})(\widehat{d}_{nt_1} - \widetilde{d}_{nt_1}) + f'_n(d^*_{nt_2})(\widehat{x}_{nt_2} - \widetilde{d}_{nt_2}) \\ &\quad + g'_{nt_2}(d^*_{nt_2})(\widetilde{d}_{nt_1} - \widehat{d}_{nt_1} + \widetilde{d}_{nt_2} - \widehat{x}_{nt_2}) \\ &\quad + \beta^*_{\diamond t_2}(\widehat{x}_{nt_2} - \widetilde{d}_{nt_2}) \quad (68a)\\ &= g'_{nt_1}(\widehat{d}^*_{nt_1})(\widehat{d}_{nt_1} - \widetilde{d}_{nt_1}) + g'_{nt_2}(d^*_{nt_2})(\widetilde{d}_{nt_1} - \widehat{d}_{nt_1}) \quad (68b)\\ &> 0, \quad (68c)\end{aligned}$$

where (68a) holds by the convexity of $g_{nt}(\cdot)$ together with (27a), the convexity of $f_n(\cdot)$ together with $\widehat{x}_{nt_2} > \widetilde{x}_{nt_2}$ and (55a), and the concavity of $-\delta_n(\sum_{t\in\mathcal{T}} x_{nt} - \Gamma_n)^2$ together with Lemma A.1, recalling that $\widehat{\boldsymbol{b}}_n, \widetilde{\boldsymbol{b}}_n \in \mathcal{B}_n(A)$ with $\sum_{t\in\mathcal{T}} d_{nt} = A < \sum_{t\in\mathcal{T}} d^*_{nt}$; (68b) holds by (10) in Lemma 3.2 together with (25); and (68c) follows from (61b).

If $\widehat{x}_{nt_2} < \widetilde{x}_{nt_2} = \widetilde{d}_{nt_2}$, Δu_n is given by,

$$\begin{aligned}\Delta u_n &> g'_{nt_1}(\widehat{d}^*_{nt_1})(\widehat{d}_{nt_1} - \widetilde{d}_{nt_1}) + f'_n(\widetilde{d}_{nt_2})(\widehat{x}_{nt_2} - \widetilde{d}_{nt_2}) \\ &\quad + g'_{nt_2}(d^*_{nt_2})(\widetilde{d}_{nt_1} - \widehat{d}_{nt_1}) + \beta^*_{\diamond t_2}(\widehat{x}_{nt_2} - \widetilde{d}_{nt_2}) \\ &\quad + 2\delta_n(\Gamma_n - \sum_{t\in\mathcal{T}} \widetilde{x}_{nt})(\widetilde{d}_{nt_2} - \widehat{x}_{nt_2}) \quad (69a)\\ &> f'_n(\widetilde{d}_{nt_2})(\widehat{x}_{nt_2} - \widetilde{d}_{nt_2}) + \beta^*_{\diamond t_2}(\widehat{x}_{nt_2} - \widetilde{d}_{nt_2}) \\ &\quad + 2\delta_n(\Gamma_n - \sum_{t\in\mathcal{T}} \widetilde{x}_{nt})(\widetilde{d}_{nt_2} - \widehat{x}_{nt_2}) \quad (69b)\\ &> 0, \quad (69c)\end{aligned}$$

where (69a) holds by the convexity of $g_{nt}(\cdot)$ together with (27a), the convexity of $f_n(\cdot)$ together with (55a), and the concavity of $-\delta_n(\sum_{t\in\mathcal{T}} x_{nt} - \Gamma_n)^2$ together with Lemma A.1, recalling that $\widehat{\boldsymbol{b}}_n, \widetilde{\boldsymbol{b}}_n \in \mathcal{B}_n(A)$ with $\sum_{t\in\mathcal{T}} d_{nt} = A < \sum_{t\in\mathcal{T}} d^*_{nt}$; (69b) uses (68b); and (69c) uses (66) together with $\widetilde{\beta}_{nt_2} > \beta^*_{\diamond t_2}$.

Case IV.2, $\widetilde{d}_{nt_2} \ge \sum_{k\in\mathcal{N}} d^*_{kt_2}$*:* Using the same argument as in Case III, (55b) is applicable. Then similar to the analysis of Case IV.1, $\Delta u_n > 0$.

Case V, $\widehat{d}_{nt_2}, \widetilde{d}_{nt_2} \in \mathcal{R}_3$

In this case, $\widehat{\beta}_{nt_2} < \widetilde{\beta}_{nt_2} < \beta^*_{nt_2}$, so (8) ensures that $\widehat{x}_{mt_2} = \widetilde{x}_{mt_2} = d^*_{mt_2}$ for all $m \in \mathcal{N} \setminus \{n\}$, and $\widehat{x}_{nt_2} \le \widetilde{x}_{nt_2} < d^*_{nt_2}$.[4] Hence, (51) becomes,

$$\begin{aligned}&\Delta\tau_n = \\ &\quad c_t(D_{t_1} + \sum_{m\neq n} d^*_{mt_1} + \widetilde{d}_{nt_1}) - c_t(D_{t_1} + \sum_{m\neq n} d^*_{mt_1} + \widehat{d}_{nt_1}) \\ &+ c_t(D_{t_2} + \sum_{m\neq n} d^*_{mt_2} + \widetilde{x}_{nt_2}) - c_t(D_{t_2} + \sum_{m\neq n} d^*_{mt_2} + \widehat{x}_{nt_2}). \quad (70)\end{aligned}$$

Using (52) and (70) in (53) gives,

$$\begin{aligned}\Delta u_n &= g_{nt_1}(\widehat{d}_{nt_1}) - g_{nt_1}(\widetilde{d}_{nt_1}) + g_{nt_2}(\widehat{x}_{nt_2}) - g_{nt_2}(\widetilde{x}_{nt_2}) \\ &\quad - \delta_n\Big(\sum_{t\in\mathcal{T}} \widetilde{x}_{nt} - \Gamma_n\Big)^2 + \delta_n\Big(\sum_{t\in\mathcal{T}} \widehat{x}_{nt} - \Gamma_n\Big)^2 \\ &> -\mu\left(\widetilde{d}_{nt_1} - \widehat{d}_{nt_1}\right) - g'_{nt_2}(d^*_{nt_2})\left(\widetilde{x}_{nt_2} - \widehat{x}_{nt_2}\right) \\ &\quad - \delta_n\Big(\sum_{t\in\mathcal{T}} \widetilde{x}_{nt} - \Gamma_n\Big)^2 + \delta_n\Big(\sum_{t\in\mathcal{T}} \widehat{x}_{nt} - \Gamma_n\Big)^2, \quad (71)\end{aligned}$$

where the inequality holds by the convexity of $g_{nt}(\cdot)$ together with (27a) for the first term, and with $\widehat{x}_{nt_2} \le \widetilde{x}_{nt_2} < d^*_{nt_2}$ for the second term.

Using (62) from Lemma A.1 together with (27a), the concavity of $-\delta_n(\sum_{t\in\mathcal{T}} x_{nt} - \Gamma_n)^2$, and $\widetilde{x}_{nt_2} > \widehat{x}_{nt_2}$ gives,

$$\begin{aligned}&-\delta_n\Big(\sum_{t\in\mathcal{T}} \widetilde{x}_{nt} - \Gamma_n\Big)^2 + \delta_n\Big(\sum_{t\in\mathcal{T}} \widehat{x}_{nt} - \Gamma_n\Big)^2 \\ &\qquad > g'_{nt_2}(d^*_{nt_2})\left(\widetilde{d}_{nt_1} - \widehat{d}_{nt_1} + \widetilde{x}_{nt_2} - \widehat{x}_{nt_2}\right) \\ &\qquad > \mu\left(\widetilde{d}_{nt_1} - \widehat{d}_{nt_1}\right) + g'_{nt_2}(d^*_{nt_2})\left(\widetilde{x}_{nt_2} - \widehat{x}_{nt_2}\right). \quad (72)\end{aligned}$$

Thus, it follows from (71) and (72) that $\Delta u_n > 0$ whenever $\widehat{d}_{nt_2}, \widetilde{d}_{nt_2} \in \mathcal{R}_3$.

In summary, the analysis presented in Cases I-V shows that inequality (28) holds for all $\widehat{d}_{nt_2} \ge d^*_{nt_2}$. *End of proof.*

[4]The equality $\widehat{x}_{nt_2} = \widetilde{x}_{nt_2} = 0$ can occur if $c'_t(D_{t_2} + \sum_{m\neq n} d^*_{mt_2}) \ge \widetilde{\beta}_{nt_2} > \widehat{\beta}_{nt_2}$.